

Pressure Drop Correlation for Pipeline Flow of Solid-Liquid Suspensions

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SCOPE

The principal aim of this work was to develop means for prediction of pressure drops for the flow of solid-liquid suspensions in pipelines. The heterogeneous suspensions encountered in hydraulic transport of solids usually have a very strong tendency for settling. Indeed, for most of these systems this tendency is so pronounced that it is virtually impossible to obtain uniform distributions of the solids through the agitation caused by the flow within the pipeline or, for that matter, by means of supplementary mixing within vessels. The number of variables which govern this type of flow is quite large, and the hydrodynamic problem relating to suspension flow, in general, is so complex that all theoretical solution attempts up to now have left it virtually intact.

Suspended solids flow in pipelines is of interest because it is applicable to problems relating to the handling and transportation of solid raw materials and products, solid wastes, and sludges. Largely because the variety of suspension systems is almost unlimited, published reports in this area pertain to studies which are, necessarily, circumscribed in some manner. Clearly, it would be expedient to study only, or at least primarily, those suspension systems of particular importance to the individual researcher, and this, to a great extent, is the type of work which prevails in the literature on slurry flow. Consequently, in many of the available works the solids used do not possess well-defined sizes and shapes, thereby making it quite difficult to evaluate the effects of these two important variables on the flow.

A primarily experimental approach to the problem is inevitable because of the apparently hopeless prospects for achieving a mathematical solution to the general hydrodynamic problem. However, an experimental study is burdened by some rather difficult constraints arising mainly from the following three characteristics of the problem:

the abundance of variables which govern the flow, the vast ranges over which most of the variables may vary, and the inherent limitations on accuracy and reproducibility of the data which can be obtained with these heterogeneous settling slurries. The last one of these is quite crucial, and it clearly resides in the fact that one is really dealing with an unstable system. In many suspensions of practical interest, the settling effect is so pronounced, indeed often so predominant, that marked variations in the data, from experiments conducted under apparently identical conditions, are observed, although some properties of the flow, among them the pressure drop, are not so strongly affected by this.

The need to predict pressure drops for flowing slurries is an eminently practical one and represents the primary motivation for this work. Because of the vastness of the problem, however, it was necessary to make some compromises. The major concession in the present study was, no doubt, the exclusion from consideration of slurries consisting of mixed sizes of suspended particles, which, of course, abound in practice. The effect of particle size distribution on slurry flow behavior is, to a large extent, still a mystery, but we know that the settling characteristics can be altered, quite dramatically sometimes, through appropriate formulation of the solid mixture. The full promise of hydraulic transport as a realistic alternative to traditional methods of solids transportation can perhaps be realized only when this aspect of the problem is studied and understood. The unappraised use of some mean value of the particle sizes as a device for adapting correlations, based on isodimensional suspended particle data, to mixed size slurries must, under the circumstances, be viewed as merely a necessary expedient. The role of particle shape perhaps ranks equally with that of particle size distribution.

SUMMARY

The slurry flow correlation established in this work is represented by the relations

$$f = f_w + 0.3202\xi \quad \text{for } \xi \geq 1$$

$$f = f_w + 0.3202\xi^{1.358} \quad \text{for } \xi < 1$$

where

$$\xi = C^{0.4925} f_w^{0.5586} C_D^{-0.1095} \left\{ \frac{g(s-1)D}{V^2} \right\}^{0.9002}$$

This correlation was based on a collection of 1,511 data points. Taken together, the data points pertained to the flow in 2-, 1-, and 1/2-in. pipelines of water suspensions of 12 different sizes of solid particles ranging from 31μ to $4,380\mu$ and covering, in round figures, suspended solids densities of 2.3, 3.0, 4.4, 7.5, and 11.3 g./cc. For the entire set of 1,511 data points, the correlation predicts the slurry friction factor with an absolute average deviation of

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9.4%, with deviations in 76 points exceeding 30% and in only 9 points exceeding 50%.

The predictive effectiveness of our slurry flow correlation is satisfactory for design purposes, particularly in view of the fact that, even for flow in the absence of solids (a much simpler, and a considerably more intensively researched problem), available design methods cannot, as they should not, claim predictive tolerances within less than 5 to 10%. It might appear that we have examined a reasonably broad range of the pertinent variables in slurry flow, but this must be viewed within the overall context of the problem. The variety of suspended systems encountered in practice is enormous.

In view of the present status of knowledge in the area of suspension flow (against which our work must be measured), a meaningful assessment of our contribution cannot be presented without citing the preeminent contributions of Durand and his coworkers (9, 11, 12, and 13). In terms particularly of the range of variables considered, Durand's work is unsurpassed. His slurry flow correlation is given by

$$f = f_w + (\text{const.}) f_w C \left\{ \frac{V^2}{gD} \sqrt{C_D} \right\}^{-1.5}$$

and predicts our own data with an absolute average deviation of 18.0%, with 288 and 163 data points having devia-

tions exceeding 30% and 50%, respectively. This is by no means the first demonstration of the limitations of Durand's correlation, a comparison not intended to diminish the stature of his contribution, which is neither necessary nor possible. Durand and his group used mainly sands and gravels, materials which were quite suitable, considering both the massive scale of their experiments and their interest in applications to dredging. Besides these considerations, Durand appealed to some extent to a presumed model of the flow in deriving his correlation, as might be surmised from its surprisingly simple form.

In our own work we chose to dispense with models entirely, forfeiting both their advantages and their constraints. In fact, the arguments in our own work were intended merely to provide justifications for inclusion of the various variables in our correlation and perhaps also to indicate a logical approach outside the confines of a model. Therefore the weights assigned to the different variables in our correlation were established on the basis of the experimental data exclusively, precluding the need for reliance on a possibly uncertain model. We have, in addition, developed supplementary correlations relating to flow in the absence of solids and also to the free settling of particles. These eliminate the need for trial and error calculations, which use of our correlation or of Durand's would otherwise entail. These latter results conform to the practical character of our work.

PREVIOUS WORK

The large body of published work relating to suspension transport includes basic studies typified by those of Goldsmith and Mason (17, 18), Happel and Byrne (20), and Happel and Brenner (19). These basic studies are concerned with particle-fluid and particle-particle interactions in single-, double-, or multiple-sphere or rod suspensions. A basic and mathematically rigorous treatment can be successfully applied to these types of highly idealized "dilute" suspensions, provided some rather severe restrictions on the flow conditions are imposed. Such studies are of intrinsic value, of course, and may be useful in some practical situations. Within the context of hydraulic transport of solids, however, they represent limiting situations if we choose to view them in this light, too far removed from the actual situation to be of practical consequence.

Dispersive Stress

A somewhat basic approach which attempts to relate the flow to measurable internal parameters related to the prevailing momentum transfer processes is based upon the researches of Bagnold (2 to 4) and his concept of the dispersive stress. The approach is phenomenological, depending upon the experimental determination of the so-called dispersive stress. Bagnold demonstrated that, by shearing a concentrated suspension of neutrally buoyant spheres in an annular region, a dispersive stress, which could be correlated in terms of the variables of the system, existed between the particles.

The approach was utilized to some extent by Daniel (10) in his study of the flow of suspensions in a rectangular channel. The experimental determination of the dispersive stress parameter is precluded in suspensions with

a strong tendency to settle because it is exceedingly difficult to maintain a uniform distribution of solids. Moreover, this approach is of marginal value in hydraulic transport of solids because of the difficulties of applying it to circular conduits.

Effective Viscosity

Effective viscosity correlations for suspensions of spheres and other *iso*-dimensional particles, such as those proposed in the studies of Rutgers (33), Kynch (23), and Ting and Luebberts (36), represent mainly theoretical or empirical extensions of Einstein's theoretical expression (14) for the viscosity of very dilute suspensions of noninteracting spheres in Newtonian fluids. The application of the effective viscosity concept in the treatment of the pipeline flow of settling suspensions entails merely a recasting of the actual pressure drop-flow rate data into a different form, an unnecessarily cumbersome and round-about procedure with no demonstrable merit. The implied promise in the effective viscosity concept is that it will reduce the treatment of the suspension flow problem to that of a simple Newtonian, or at least homogeneous, fluid. That is, given the effective viscosity for a suspension, the correlations developed for simple homogeneous Newtonian fluids (or non-Newtonian fluids) can be used.

The procedure, therefore, depends crucially upon the possibility of making meaningful (in the restricted sense just stated) independent viscometric measurements, a rather difficult trick with the strongly settling, manifestly heterogeneous suspensions of practical interest. Nonetheless, effective viscosity correlations are quite useful for the purposes they have been used, which have rarely, if ever, included the correlation of the flow behavior of settling suspensions in pipelines.

Continuum Approach

Generally, when the maximum particle size in the suspension is quite small and the flow velocity is sufficiently high, the distribution of solids across the pipe is more or less uniform, and the continuum approach is applied. For the flow of water suspensions of thoria and kaolin in pipes, Thomas (35) used the Bingham plastic model in establishing his correlations, but the largest mean particle size was about 13μ . The use of a continuum non-Newtonian approach is predicated upon the ability to provide a rheological characterization of the suspension through suitable viscometric means. Some rather important problems relating to the correlation of pressure drop-flow rate data in non-Newtonian fluids remain unresolved, and particularly those arising from ambiguities surrounding the definition of a suitable generalized Reynolds number are most persistent. Nonetheless, useful correlation procedures, based largely on data for aqueous, essentially homogeneous, high polymer solutions have been proposed (25 and 26).

Interestingly, the observation that, in some suspended flows at sufficiently high velocities, the solid material is concentrated within the central core with an outer annular region of clear liquid near the pipe wall, led many investigators to presume that Bingham plastic flow prevailed. This type of behavior is generally common in paper pulp suspensions. Its connection with Bingham plastic flow rests upon the fact that, in laminar pipe flow of this ideal, essentially fictitious, material, there is always a central elastic plug surrounded by an annular region of liquid. The possible relationship between the two flows is quite deceptive, of course, because the suspension is not a continuum. The continuum non-Newtonian approach has been moderately successful with suspensions of fines which are frequently viewed as plastic materials with a yield stress (6 and 38).

Hydraulic Transport

Research on hydraulic transport has had a long history, and interest in it has endured and grown. Over a half a century ago Blatch (21) proposed the relation

$$h_m = h_w + AC \quad (1)$$

where h_m and h_w are the head losses (in feet of water per foot of pipe) for the mixture and for the suspending liquid at the same flow rate, respectively, and C is the mean solids concentration. In Equation (1) A was assumed constant, but Blatch, who took data using 1-in. brass and galvanized pipe and suspensions of -20 to +40-mesh and -60 + 100-mesh sand, found that the value of A depended on solid size. Later Howard (22), using 0.4-mm. sand in a 4-in. pipe, showed that A depended on pipe diameter as well.

Solids Suspension Flow

Based on the idea of summing the work needed to maintain the solids in suspension, together with the energy dissipation for the flowing water, Wilson (39) derived the relation

$$h_m = h_w + A' \frac{v_*}{V} C \quad (2)$$

where v_* is the free-falling velocity of the particle and V is the mean suspension velocity. Wilson's treatment does not account for interactions among the various energy contributions in the flow. Newitt et al. (29) conducted experiments with solids of sizes from -240 mesh to 3/16

in. and a specific gravity between 1.18 and 4.6 in a 1-in. pipe. Using Wilson's approach they proposed the following relations for

homogeneous flow: $V \geq V_H = (1800 g D v_*)^{1/3}$

$$\frac{i - i_w}{i_w C} = 0.6(s - 1) \quad (3)$$

heterogeneous flow: $V_B < V < V_H$

$$\frac{i - i_w}{i_w C} = 1100 \frac{g D}{V^2} \frac{v_*}{V} (s - 1) \quad (4)$$

moving bed flow: $V \leq V_B = 17v_*$

$$\frac{i - i_w}{i_w C} = 66 \frac{g D}{V^2} (s - 1) \quad (5)$$

In Equations (3) to (5) i and i_w are the head losses for the mixture and water at the same mean velocity, respectively; C is the discharge concentration of solids; and s is the ratio of solid to liquid densities. The transition velocities V_H and V_B used to delineate the flow regimes depicted are given by

$$V_H = (1800 g D v_*)^{1/3} \quad (6)$$

and

$$V_B = 17v_* \quad (7)$$

Equation (6) is simply derived by equating Equations (3) and (4), and Equation (7) is similarly derived by equating Equations (4) and (5).

In the area of solids suspension flow, Durand and his coworkers (9 to 13) revealed the results of a study so extensive that it exceeded, or at least equaled, the combined effort of all other researchers in the early fifties. The Durand group studied sand and gravel suspensions with particle sizes ranging from 0.2 to 25 mm.; pipe diameters from 1½ to 23 in.; and solids concentrations up to 60% by weight. They proposed the following criteria for delineating the main flow regimes observed in suspended solids flow.

For $d < 30\mu$: homogeneous flow, i.e., a uniform distribution of solids in flow exhibiting plastic behavior

For $30\mu < d < 50\mu$: intermediate flow, i.e., depending upon the suspension velocity either homogeneous or heterogeneous flow can occur.

For $d > 50\mu$: heterogeneous flow

(a) For $50\mu < d < 0.2$ mm.: suspended flow

(b) For 0.2 mm. $< d < 2$ mm.: transition flow

(c) For $d > 2$ mm.: saltation flow

Despite the fact that the foregoing criteria have been confirmed by others (for the same solids) the particle size alone is clearly entirely too inadequate a measure of the nature of the flow regime which prevails.

A principal result of the work by Durand's group is the correlation

$$\frac{i - i_w}{i_w C} = K \left\{ \frac{V^2}{g D} \sqrt{C_D} \right\}^{-1.5} \quad (8)$$

where C_D is the drag coefficient for the free-falling particle, assumed spherical and of diameter d , given by

$$C_D = \frac{4}{3} \frac{g d (s - 1)}{v_*^2} \quad (9)$$

Based on a very large number of tests on sand slurries, the constant K in Equation (8) was found to be 318. To account for the effect of particle density, moreover, a modification of Equation (8) is given by

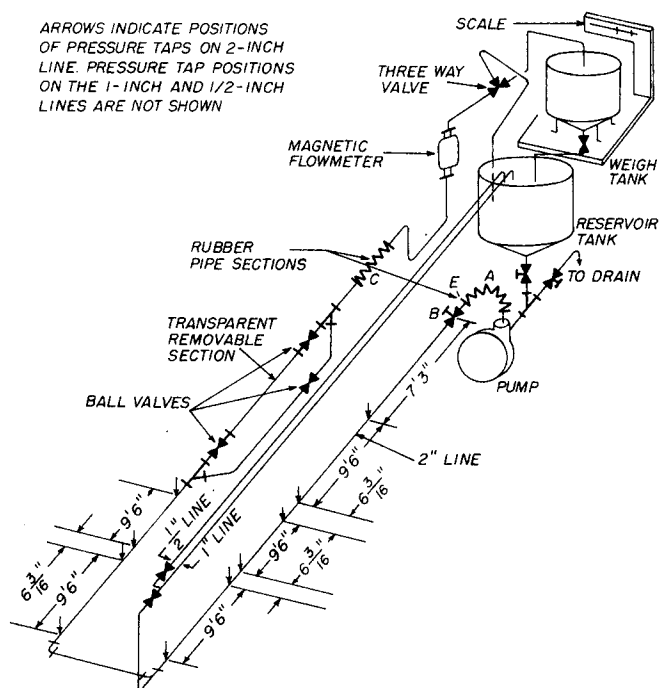


Fig. 1. Schematic representation of equipment.

$$\frac{i - i_w}{i_w C} = \bar{K} \left\{ \frac{V^2}{gD(s-1)} \sqrt{C_D} \right\}^{-1.5} \quad (10)$$

For the sand slurries Durand's group used $(s-1) = 1.65$, and, obviously, by comparing Equations (8) and (10) $\bar{K} = 150$.

Critical Velocity of Slurries

A further result of Durand's work is an empirical equation for predicting the critical velocity of the slurry, i.e., the velocity below which a stationary deposit of solids forms in the pipe. The critical velocity correlation is given by

$$V_c = F_L \sqrt{2gD(s-1)} \quad (11)$$

where F_L is a dimensionless function of particle diameter given by Durand. The result in Equation (11) is of considerable practical value, for the critical velocity, if sufficiently high in a given case, may be the limiting parameter.

Ellis and Round (15) took data using nickel ($s = 8.90$) slurries with particle size of 106μ in a 1-in. pipe. In order to get a satisfactory description of their data using Equation (10), they found that it was necessary to change the value of \bar{K} and also the exponent -1.5 . That a correlation based on so extensive a piece of research work, as that of Durand and his coworkers, should fail to describe these data is testimony to the magnitude of the research problem and the abundance of the variables which govern it.

Other interesting work includes Worster and Denny's (40) who used 0.1- to 1-in. gravel and coal slurries in 3-, 4-, and 6-in. pipe. Smith (34) took data in slurries with mixed particle size in 2- and 3-in. pipe. In using Durand's relation (8), Smith recommends using

$$d_e = \sum_i w_i / \sum_i (w_i / d_i) \quad (12)$$

for calculation of an equivalent particle diameter d_e . In Equation (12) w_i is the weight fraction of particles of diameter d_i . Miller and Cloete (8 and 27) and Oedjoe et al. (5, 30 to 32) report pumping sand slurries at extremely high concentrations, intermediate between those of freely settled and vibration-packed beds of these solids. Among the rare works in which liquids other than water have been used is that of Gay (16) who suspended particles of alumina, nickel, copper, and glass in sodium, xylene, and glycerine in small-scale laboratory equipment. The use of liquids other than water in pilot-size pipelines, a scale which ultimately must be used in this type of research, is somewhat impractical.

EXPERIMENTAL WORK

The slurry flow equipment used in our experiments is shown schematically in Figure 1. It consisted of a 160-gal. conical bottom stainless steel reservoir connected to a slurry-type centrifugal pump with a 3-in. suction and 2-in. discharge. The pump was connected to a 5 hp. solid state controlled induction motor, which could maintain constant speed using the signal from a tachometer control, a pressure transducer at the pump discharge, or the signal from a magnetic flowmeter. Test pipes of nominal diameters of 2, 1, and $\frac{1}{2}$ in. were used. The overall lengths of straight test sections were approximately 35-ft. each. A 2-in. magnetic flowmeter was connected to the return leg of the main 2-in. test loop. Flow rates could, therefore, be determined using the calibrated magnetic flowmeter or an 80-gal. stainless steel weigh tank mounted on a scale above the reservoir.

In our slurry flow experiments we determined flow rate, pressure drop through straight sections of the test pipes, and the discharge concentrations of the solids. The slurries consisted

TABLE 1. PROPERTIES OF GLASS BEADS USED IN SLURRY FLOW EXPERIMENTS IN THIS WORK

Average diameter (μ)	Density (g./cc.)
90	4.434
505	4.430
1,015	4.400
475	2.486
1,340	2.977
4,380	2.964

TABLE 2. SOLID PARTICLES USED IN SUSPENSION FLOW EXPERIMENTS IN $\frac{1}{2}$ -IN. PIPELINE BY MURPHY, YOUNG, AND BURIAN (28)

Material	Diameter (μ)	Density g./cc.	Max. discharge conc. (wt. %)	No. Data*
Glass	31.0	2.344	10.0	6
	50.8	2.344	27.5	14
	66.0	2.790	34.8	81
	289.6	2.851	48.5	155
	797.6	2.504	59.6	86
Steel	378.5	7.503	47.8	71
	1834	7.503	30.5	46
Lead	1283	11.30	63.8	172

* Total No. = 631.

of glass beads suspended in water. Spheres made from two different glasses, having different densities, were used. The properties of the glass beads are listed in Table 1. Complete details relating to the equipment and procedure, and the experimental data themselves have been documented elsewhere (37).

RESULTS AND CORRELATIONS

Slurry Flow Data

The total number of flow data in the 2-, 1-, and 1/2-in. pipelines that we took in this work was 917. In addition, we used the data of Murphy, Young, and Burian (28) who used a 1/2-in. pipeline with only the slurries listed in Table 2. We used only 594 of their 631 data points in establishing our slurry flow correlation because, for the remaining 37 data points, the observed pressure loss was lower than that for water. For these data the differences in friction factors for the slurry and the water were negative, which, aside from indicating possible experimental error, precluded their use in the slurry correlation we developed. The total collection of data used, therefore, consisted of 1,511 data points. Altogether these data relate to water suspensions of 12 different sizes of solid particles ranging from 31 μ to 4,380 μ and covering, in round figures, densities of 2.3, 3.0, 4.4, 7.5, and 11.3 g./cc. and pipe diameters of 2, 1, and 1/2 in.

Free Settling Velocity of Spheres

In order to use the slurry flow correlations developed, the drag coefficient for the free settling of spheres must be known. The flow past a sphere is a classical problem with an extensive literature. In a recently completed study in this laboratory relating to settling of spheres in macromolecular solutions, Carey (7) made extensive measurements of the terminal velocities of spheres in Newtonian fluids. In addition, Carey made a comprehensive search and a critical examination of published data on settling of spheres in Newtonian fluids and selected the data of Bacon (1) and McPherson (24) for inclusion in a data store used as the basis of the drag coefficient correlation for settling of spheres. Like his own data, Bacon's and McPherson's conformed to the rigid requirements on control of experimental conditions and consideration of the wall effect. The entire set of data, corrected for wall effect, was comprised of 210 points and covered the range of Reynolds numbers $8.5 \times 10^{-8} \leq N_{Re} \leq 1.5 \times 10^5$ with the Reynolds number defined by

$$N_{Re} = \frac{d\rho v_w}{\mu} \quad (13)$$

where μ is the viscosity of the liquid. The drag coefficient correlation obtained by Carey is

$$C_D = \left\{ \left(\frac{24}{N_{Re}} \right)^{1/2} + 0.34035 \left[N_{Re}^{0.06071} + \frac{1}{1.72013 + 0.018 N_{Re}} \right] \right\}^2 \quad (14)$$

Equation (14) is valid for $N_{Re} \leq 1.5 \times 10^5$. For 210 data points used in developing Equation (14), the absolute average deviation in the value of C_D is 3.73%, with a maximum deviation of 19.88% for the point $N_{Re} = 1.275 \times 10^5$. Furthermore, of the total number of 210 data points the deviations for only 11 points exceed 10%, with the remaining 199 points having deviations below 10%. Thus Equation (14) describes the free settling sphere data to a level of accuracy considerably surpassing that which

may be required for use in the slurry flow correlations, and it, moreover, is based on the most carefully scrutinized falling sphere data to date.

Because the settling velocity v_w is unknown, a trial and error procedure must be used to evaluate C_D from Equation (14). To preclude this we have developed the correlation

$$\begin{aligned} \log_{10} N_{Re} = & -1.38 + 1.94 \log_{10} \Lambda \\ & - 8.60 \times 10^{-2} (\log_{10} \Lambda)^2 - 2.52 \times 10^{-2} (\log_{10} \Lambda)^3 \\ & + 9.19 \times 10^{-4} (\log_{10} \Lambda)^4 + 5.35 \times 10^{-4} (\log_{10} \Lambda)^5 \end{aligned} \quad (15)$$

in which

$$\Lambda = N_{Re} C_D^{1/2} = \left\{ \frac{4}{3} \frac{gd^3\rho(\rho_s - \rho)}{\mu^2} \right\}^{1/2} \quad (16)$$

and it, therefore, does not contain v_w . Equation (15) is valid up to $N_{Re} \leq 1.5 \times 10^5$. In terms of the definition

$$\% \text{ deviation} = \frac{C_{D \text{ calc.}} - C_{D \text{ exp.}}}{C_{D \text{ calc.}}} \times 100 \quad (17)$$

the absolute average deviation for all 210 corrected falling sphere data points is 3.12% with a maximum deviation of 12.87%. Equation (15) will be used in calculations with the slurry flow correlation to be presented later.

Pipe Flow in Absence of Solids

Homogeneous liquid pipe flow correlations, analogous to those obtained in the preceding section for falling spheres, would permit straightforward calculations to be possible with the slurry flow correlations to be presented in the next section. For fully developed laminar flow, the Hagen-Poiseuille law applies. For the more prevalent case of turbulent flow, the Blasius equation

$$f_w = 0.0791 (N_{Rew})^{-1/4} \quad (18)$$

may be used. In Equation (18) the friction factor f_w and the Reynolds number N_{Rew} relating to the homogeneous liquid (water) are defined by

$$f_w = \frac{\Delta p_w}{2\rho V^2} \quad (19)$$

$$N_{Rew} = \frac{D\rho V}{\mu} \quad (20)$$

where Δp_w is the pressure drop observed when the homogeneous suspending liquid is flowing in a circular pipe of diameter D and length L at the velocity V . ρ and μ are the density and viscosity of the liquid, respectively.

Blasius' Equation (18) is approximate and, in order to get a better description of the $f_w(N_{Rew})$ relation for the pipes used in our slurry flow studies, we developed alternative expressions which gave closer fits to the data, as follows:

$$\text{2-in. pipe: } \log_{10} f_w = -1.306 - 0.2044 \log_{10} N_{Rew} \quad (21)$$

$$\text{1-in. pipe: } \log_{10} f_w = -0.8395 - 0.2899 \log_{10} N_{Rew} \quad (22)$$

$$\text{1/2-in. pipe: } \log_{10} f_w = -0.9912 - 0.2640 \log_{10} N_{Rew} \quad (23)$$

Equations (21), (22), and (23) describe our water flow data with deviations not exceeding 1.5%. In addition to our own data, we used those of Murphy, Young, and Burian (28). They used 1/2-in. pipe and their water data can be represented by

$$\log_{10} f_w = -0.5201 - 0.5137 \log_{10} N_{Re w} + 0.02942 (\log_{10} N_{Re w})^2 \quad (24)$$

Equation (24) describes the 1/2-in. water data of Murphy et al. with an absolute average deviation of 0.82% and a maximum deviation of 3.51%.

There are other expressions for describing $f_w(N_{Re w})$ or the equivalent relation for homogeneous pipe flow; the most famous is Prandtl's resistance law

$$\frac{1}{\sqrt{f_w}} = 4.0 \log_{10} (N_{Re w} \sqrt{f_w}) - 0.40 \quad (25)$$

Because the problem relating to pipe flow in the absence of suspended matter has been studied exhaustively, we will not dwell further upon it here. Our aim is to describe the particular empirical expressions for water flow that we will be using in subsequent analyses; the foregoing results fulfill this objective.

Slurry Flow Correlation

Under isothermal conditions at least 12 variables are needed to describe suspension flow behavior, provided that pipeline roughness and solids density distribution (for solids of mixed densities) are excluded from consideration. These variables are shown in Equation (26) and are defined in the Notation.

The total number of 12 variables can be reduced to 9 independent dimensionless groups which can be used to display their functional dependence. One way of expressing this is through a relation of the type

$$f = \frac{\Delta p}{2 \rho V^2} \frac{D}{L} = \phi \left[\frac{D \rho V}{\mu}, \frac{d \rho V}{\mu}, \frac{L}{D}, \frac{\rho_s}{\rho}, \frac{V^2}{gD}, C, \psi, \epsilon \right] \quad (26)$$

Under some conditions, certain simplifications can be made in Equation (26). For example, if the pipe length is large the dependence on the (L/D) ratio becomes insignificant, and this term can be dropped. The entrance length requirements, unlike the case for flow in the absence of solids, are not known for slurry flows, and their possible influence must be ascertained by measurement of axial variations in pressure drop. A few other simplifications in the dependence depicted by Equation (26) will accrue upon imposition of various restrictions, and these include the dropping of the dependence on ψ for isodimensional particles, and ϵ for spherical solids. For fully developed pipe flow in the absence of suspended materials, all but the first dimensionless group on the right-hand side of Equation (26) can be dropped. For fully developed pipe flow of a suspension composed of identical spherical solid particles, the dimensionless groups which can be dropped are (L/D) , ψ , and ϵ ; the possible dependence on the so-called Froude number (V^2/gD) must, unlike the case in the absence of solids, be retained. Accordingly, for this case Equation (26) takes the form

$$f = \frac{\Delta p}{2 \rho V^2} \frac{D}{L} = \phi \left[\frac{D \rho V}{\mu}, \frac{d \rho V}{\mu}, \frac{\rho_s}{\rho}, \frac{V^2}{gD}, C \right] \quad (27)$$

Despite the substantial reduction in the number of variables in going from Equation (26) to Equation (27), the functional dependence is still unknown. A purely experimental approach designed to establish the relationship stated in Equation (27), even for modest ranges of the dimensionless groups appearing, would entail so massive an effort as to be impracticable. In addition, the dimensionless groups appearing on the right-hand side of Equation (27), although they are quite legitimate choices for defining the problem in a general sense, do not, on the whole, bear an obvious relationship to the friction loss. An entirely different approach is utilized in this work, and it will be described following a brief discussion of the relationship between the flow of suspended solids and the free settling velocity of a single particle.

As might have already been suspected, information relating to the free settling of single solid particles will be incorporated in the slurry flow correlation developed in this work. The free settling velocity v_s of a solid sphere of diameter d and density ρ_s in a fluid of viscosity μ and density ρ does not appear in any of the groups in Equation (26) or (27). It will be shown that the dimensionless groups in these equations are sufficient to define fully the quantity Λ given by Equation (16), and, therefore, using the results previously discussed, the drag coefficient C_D and the corresponding Reynolds number N_{Re} for this flow are also fully defined.* It will be convenient to use the definitions

$$\chi_1 = \frac{D \rho V}{\mu}; \chi_2 = \frac{d \rho V}{\mu}; \chi_3 = \frac{\rho_s}{\rho}; \chi_4 = \frac{V^2}{gD} \quad (28)$$

Using these definitions and Equation (16), Λ is given by

$$\Lambda = N_{Re} C_D^{1/2} = \left\{ \frac{4}{3} \frac{\chi_2^3}{\chi_1 \chi_4} (\chi_3 - 1) \right\}^{1/2} \quad (29)$$

Equation (29) provides the connection between the general friction-loss relationship depicted in Equation (27) and the drag coefficient-Reynolds number dependence for the single particle.

We now proceed to a discussion of the slurry flow correlation. The idea behind our approach is quite simple. The starting point is to consider the various forces involved in the pipeline flow of a slurry, and to represent these or, more correctly, quantities related to their orders of magnitude in the flow by suitable mathematical expressions. Next, we form dimensionless ratios among these various terms and assume that the friction loss factor f for the slurry is a function of these ratios and also of additional variables which are known, or presumed, to be pertinent. The final steps involve establishing a correlation by curve-fitting, which is consistent with the experimental data.

In accordance with the foregoing idea, we consider the inertial and viscous forces relating to the flow and the gravitational force on the suspended solids. We assume that the order of magnitude of the inertial force F_I and viscous force F_V are depicted by the relations

$$F_I \sim \rho V^2/D \quad (30)$$

$$F_V \sim \mu V/D^2 \quad (31)$$

where the symbol \sim is used to designate an ordering statement, no equality being implied or, for that matter, necessary. Furthermore, these relations are expressed in terms of the density ρ and the viscosity μ of the suspending liquid. It is very important to recognize that the statements depicted in Equations (30) and (31) relate to forces per unit volume, while the net force of gravity and buoyancy on the solid F_D represented by

$$F_D \sim \frac{\pi}{6} d^3 g (\rho_s - \rho) \quad (32)$$

is based on the particle volume. The force ratios are

$$F_I/F_V = \frac{D \rho V}{\mu} = N_{Re w} \quad (33)$$

* The settling velocity v_s per se cannot be defined without additional specific information inasmuch as it is dimensional.

TABLE 3. EXPERIMENTAL DATA FOR GLASS SPHERE SLURRIES FROM THIS WORK

Symbol*	Diameter (μ)	Density (g./cc.)
□	90	4.434
×	505	4.430
◇	1015	4.400
+	475	2.486
⊙	1340	2.977
△	4380	2.964

* See Figures 2 and 3.

representation given by

$$f = \frac{\Delta p}{2 \rho V^2} \frac{D}{L} = \bar{F} \left[N_{Rew}, C_D, \frac{g(s-1)D}{V^2} C \right]$$

$$= F \left[f_w(N_{Rew}), C_D, \frac{g(s-1)D}{V^2}, C \right] \quad (35)$$

In the second functional form of Equation (35), we have replaced the Reynolds number for the liquid in the absence of solids N_{Rew} by its corresponding friction factor f_w , a permissible step because N_{Rew} defines f_w completely (we exclude possible ambiguities associated with the laminar-turbulent transition) and an appropriate one because the overall relationship itself pertains to the friction factor.

We must now assume a specific functional form for Equation (35) which, in the absence of solids ($C = 0$), reduces to the value f_w . The form we chose is given by

$$f - f_w = K C^\alpha f_w^\beta C_D^\gamma \left\{ \frac{g(s-1)D}{V^2} \right\}^\delta \quad (36)$$

The constants K , α , β , γ , and δ were determined by means of nonlinear least squares using Equation (36) together with 1,511 data points. The concentration C used in this equation is the discharge concentration, and C_D was calculated using Equation (15) and the definition $C_D = \Delta^2/N_{Re}^2$. In the nonlinear least squares scheme for evaluating the constants in Equation (36), initial estimates for these constants are needed. These were provided by first determining the values for the linearized form of Equation (36), the linearization amounting to taking the logarithms of both sides of the equation.

Using the nonlinear least square adjusted values of K , α , β , γ , and δ , we define the quantity

$$\xi = C^{0.4925} f_w^{0.5586} C_D^{-0.1095} \left\{ \frac{g(s-1)D}{V^2} \right\}^{0.9002} \quad (37)$$

Equation (36) with the adjusted values of the constants is represented by the line segment labelled $(f - f_w) = K\xi$ in Figure 2.* The comparison of the line $(f - f_w) = K\xi$ with the actual data depicted in Figure 2 shows that, in the nonlinear least squares computations using the difference $(f - f_w)$ as the dependent variable, the data pertaining to the range $\xi > 1$ are weighted somewhat more heavily than those in the range $\xi < 1$.

In order to overcome this, we devised the correlation

$$f = f_w + 0.3202\xi \quad \text{for } \xi \geq 1 \quad (38)$$

* Because of the massive number of data points, Figures 2 and 3 were plotted using a Calcomp Plotter driven by an IBM 360/50 computer. For symbols and experimental data of Figures 2 and 3, see Table 3.

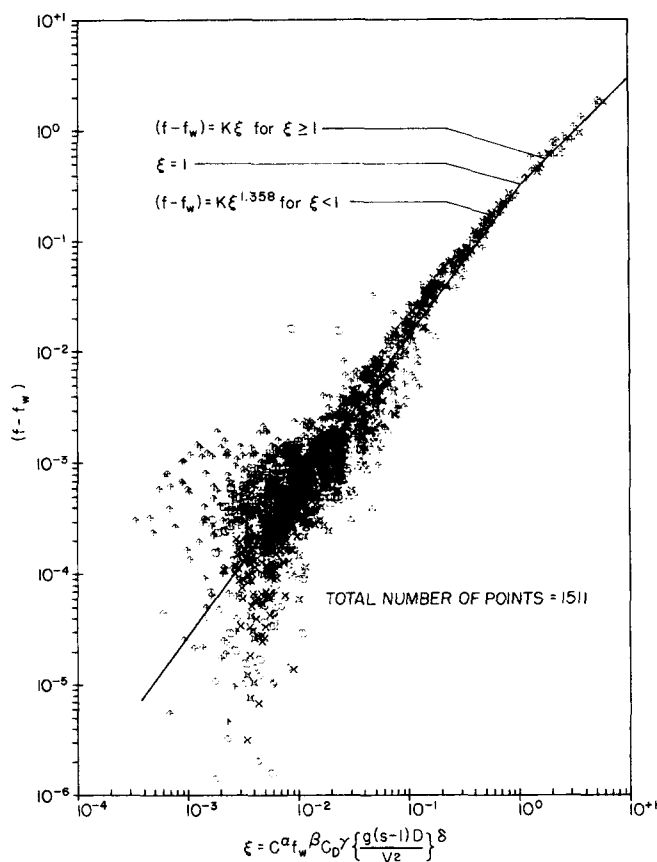


Fig. 2. Plot of $(f - f_w)$ against $\xi = C^\alpha f_w^\beta C_D^\gamma \left\{ \frac{g(s-1)D}{V^2} \right\}^\delta$ for suspension flow through 2-, 1-, and $\frac{1}{2}$ -in. pipes. For values of parameters, see Equations (36) and (37). For symbols and experimental data, see Table 3. Included are experimental data of Murphy, Young, and Burian (28) for $\frac{1}{2}$ -in. pipe; suspensions are indicated as Φ for glass beads and \boxtimes for steel and lead particles (see also Table 2).

$$F_D / \left(\frac{\pi}{6} d^3 F_I \right) = \frac{g(\rho_s - \rho)}{\rho V^2} = \frac{g(s-1)D}{V^2} \quad (34)$$

The first ratio is the Reynolds number for the pure liquid with density ρ and viscosity μ flowing at the suspension velocity V . In the second ratio we have taken into account the fact that F_I and F_D are defined relative to different bases, as previously mentioned.

We postulate that the friction factor f for the slurry depends on the dimensionless ratios (33) and (34), on the drag coefficient C_D defined in Equation (9), and on the discharge concentration C of suspended solids. The inclusion of the drag coefficient C_D for single free-falling spheres reflects our desire to account in some way for slurry settling effects. The fate of a single particle in the pipe, where it is surrounded by other particles in a shear field, is vastly different from that of a single sphere settling in a quiescent unbounded fluid. However, in the absence of knowledge pertaining to the true situation, this choice is inevitable. Moreover, it has a rather appealing feature; it is a well-defined quantity and can be readily determined. It should be emphasized that C_D is independent of the quantity defined in Equation (34). Thus, the slurry friction factor f is taken to have the functional

$$f = f_w + 0.3202\xi^{1.358} \quad \text{for } \xi < 1 \quad (39)$$

where ξ is given by Equation (37). In obtaining Equation (39), pertaining to the range $\xi < 1$, we simply drew a straight line through the center of the black mass of data for this range in Figure 2 and determined its slope. In Figure 3* we present a comparison between the correlation, Equations (38) and (39), and our own data only. The comparison does not reveal any significant difference between our data alone and the entire set shown in Figure 2. Using the definition

$$\% \text{ deviation} = \frac{f_{\text{exp.}} - f_{\text{calc.}}}{f_{\text{calc.}}} \times 100 \quad (40)$$

the correlation represented by Equation (38) and (39) gives an absolute average deviation of 9.43% for all 1,511 data points, with 76 points having deviations exceeding 30% and only 9 points having deviations exceeding 50%.

There is no discrepancy between these statements on percent deviation and the information embodied in Figures 2 and 3, although our use of differences for the ordinate value in these figures tends to convey an exaggerated impression of the deviation. By comparison, the absolute average deviation for all 1,511 data points using Durand's correlation, Equation (10), is 18.0%, with 288 data points having deviations exceeding 30% and 163 points having deviations exceeding 50%. The absolute average deviation for 1,511 points using Newitt's correlation, Equations (3) through (5), is 12.81%, with 122 points having deviations exceeding 30% and 63 points with deviations exceeding 50%.

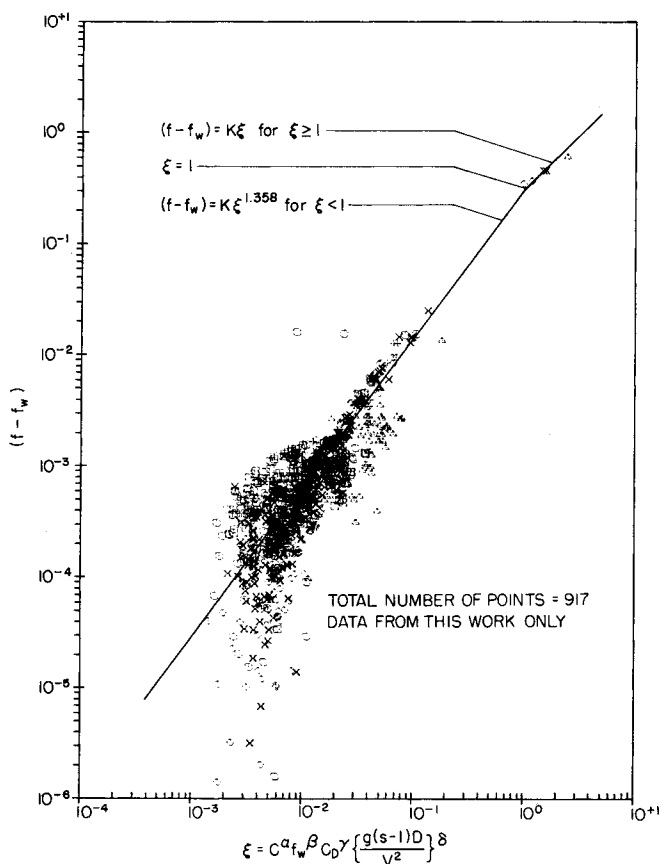


Fig. 3. Plot of (f/f_w) against $\xi = C^\alpha f_w^\beta C_D^\gamma \left\{ \frac{g(s-1)D}{v^2} \right\}^\delta$ for suspension flow through 2-, 1-, and 1/2-in. pipes. Shown are only data taken in this work. For values of parameters, see Equations (36) and (37). For symbols and experimental data, see Table 3.

DISCUSSION

The foregoing comparison between our correlation and the experimental data may suggest that we could have proceeded somewhat further in reducing the deviation. Had it been warranted, the correlation could have been extended to provide a comparatively closer agreement with all the data tested, but a rather convincing case could be made for deferring such an investment of effort pending the accumulation of more data. A strong preoccupation with merely the problem of splitting the percent deviation by successive extensions of the empirical correlation would, aside from allocating to this aspect of the problem a great deal more importance than it deserves, soon lead to diminishing returns.

In considering suspended solid flows, particularly those in which settling effects are quite pronounced, an important characteristic which emerges is that the system is unstable, in the sense described at the outset in this paper, and, therefore, there are inherent limitations on the accuracy of the data which may be obtained. Fortunately, some properties of the flow, among them the observed pressure drop, are not as terribly sensitive to effects arising from this nature of the system as, say, the discharge or spatial concentrations seem to be. The use of a broader base of data will not alter this aspect of the problem, namely, the part associated with uncertainties in the observed data arising from effects which are neither easy to detect nor to evaluate, but, at the very least, it will submerge it by the sheer force of factual evidence. Oddly enough, many authors of published research on hydraulic transport of solids are mute with respect to quantitative statements on the predictive effectiveness of their correlations. They deprive the rest of us, no doubt unwittingly, from sharing this important facet of their experiences, a sharing which might facilitate the evolution of criteria on what one could realistically expect from slurry flow design calculations.

Unlike the correlations relating to flow in the absence of solids, which relate friction loss to flow rate, the slurry flow correlation provides a relation between solids concentration and friction loss for a given flow rate. Therefore, for a given flow rate, the friction loss is indeterminate unless the concentration is specified, and vice versa. This type of relationship can, in principle, be used as the basis of optimization calculations relating to hydraulic transport of solids. Optimization calculations within the context of the present status of knowledge in this area, characterized by (besides the limitations residing in uncertainties with respect to predictive effectiveness, as discussed above) the absence of information on the important role of particle size distribution, among others, would amount to what they usually do in other problems burdened by similar circumstances: sterile mathematical exercises.

The collection of data that we have used as the basis for our correlation represents a reasonably wide range of particle sizes and densities and a rather modest range of pipe diameters, although the largest pipeline we used may be considered to be on a pilot scale. Viewed within the overall context of hydraulic transport of solids, the range of variables represented by our collection of data is far from extravagant, of course. Nevertheless, if unlimited data were available, studies of this sort would be superfluous; the aim here is to establish, on the basis of laboratory data, the means to predict the behavior on the practical scale. The total number of 1,511 data points, most of which we took ourselves, is not small by most standards of measurement. Our use of this particular set of data, covering the range of variables that it does, and the number and type of data that we took ourselves, reflects, in part, an attempt to measure ends to means.

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NOTATION

- C = discharge concentration of solids, volume %
- $C_D = \frac{4}{3} \frac{gd(\rho_s - \rho)}{\rho v_\infty^2}$, drag coefficient for free-falling sphere, dimensionless
- D = inside diameter of pipe, cm.
- d = mean particle diameter of suspended solids, cm.
- $d_e = \Sigma w_i / \Sigma (w_i / d_i)$, equivalent particle diameter (Equation 12)
- $f = (\Delta p D) / (2\rho V^2 L)$, friction factor for pipe flow of slurry, dimensionless
- $f_w = (\Delta p_w D) / (2\rho V^2 L)$, friction factor for pipe flow in absence of solids, dimensionless
- F_D = gravitational force (Equation 32), dyne
- F_I = inertial force (Equation 30), dyne/cm.³
- F_L = dimensionless function in critical velocity correlation (Equation 11)
- F_V = viscous force (Equation 31), dyne/cm.³
- g = gravitational constant, cm./sec.²
- h_m = head loss for pipe flow of slurry, in feet of water per foot of pipe
- h_w = head loss for pipe flow in absence of solids, in feet of water per foot of pipe
- i = pressure drop or friction factor for pipe flow of slurry
- i_w = pressure drop or friction factor for pipe flow in absence of solids
- K = constant
- L = pipe length, cm.
- Δp = pressure drop in length L of pipe for slurry, dyne/cm.²
- Δp_w = pressure drop in length L of pipe in absence of solids, dyne/cm.²
- $N_{Re} = d\rho v_\infty / \mu$, Reynolds number for free settling spheres, dimensionless
- $N_{Rew} = D\rho V / \mu$, Reynolds number for pipe flow in absence of solids, dimensionless
- $s = \rho_s / \rho$, ratio of solid to liquid densities, dimensionless
- V = mean velocity of mixture for slurry flow in pipeline, cm./sec.¹
- $V_B = 17v_\infty$ (Equation 7), cm./sec.¹
- V_c = critical velocity of slurry, (Equation 11), cm./sec.¹
- $V_H = (1800 g D v_\infty)^{1/3}$ (Equation 6), cm./sec.¹
- v_∞ = terminal velocity of sphere settling in an unbounded fluid, cm./sec.¹
- $K, \alpha, \beta, \gamma, \delta$ = slurry flow correlation constants obtained by nonlinear least squares adjustment (Equation 36)
- ϵ = shape factor for suspended particles, dimensionless
- $\Lambda = N_{Re} C_D^{1/2}$, dimensionless
- μ = liquid viscosity, poise
- $\xi = C^\alpha f_w^\beta C_D^\gamma [g(s-1)D/V^2]^\delta$, dimensionless
- ρ = liquid density, g./cm.³
- ρ_s = solid particle density, g./cm.³
- $\chi_1 = D\rho V / \mu$, dimensionless group (Equation 28)
- $\chi_2 = d\rho V / \mu$, dimensionless group (Equation 28)
- $\chi_3 = \rho_s / \rho$, dimensionless group (Equation 28)

- $\chi_4 = V^2 / gD$, dimensionless group (Equation 28)
- ψ = particle size distribution function, dimensionless

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